AMSC/CMSC 460 Quiz 5 , Fall 2002

1. (10) Let

$$I = \int_2^3 \int_{-1}^x x^2 \cos(xy^2)\, dy\, dx\ .$$

Given a Matlab integration function `quad('f',a,b,tol)` that computes an approximation to

$$\int_a^b f(t)dt$$

within `tol` of the true value, write code to compute an approximation within 10^{-3} of I.

(Grading: 7 points for an approximation; 10 points for achieving the error tolerance.)

Answer: We want to compute a number Q so that $|I - Q| < tol$. Now, I is the integral of the function

$$f(x) = \int_{-1}^x x^2 \cos(xy^2)\, dy$$

and we don't compute $f(x)$ exactly. Instead, we compute an approximation to it, $\tilde{f}(x)$, where $|f(x) - \tilde{f}(x)| < \delta$, where δ is the tolerance we give to `quad` when forming f. So, if we tell `quad` to integrate $\tilde{f}$ to a tolerance of ϵ, then our total error is

$$|I - Q| \le \epsilon + \int_2^3 \int_{-1}^x |f(x) - \tilde{f}(x)|\, dy\, dx \le \epsilon + \delta \int_2^3 \int_{-1}^x dy\, dx\ .$$

Any choice of ϵ and δ that keep this number less than `tol` is fine. For instance, $\epsilon = \delta = 10^{-4}$, works, but is a little conservative.

```
I = quad('f',2,3,1.e-4);

function a = f(x)
global xx
xx = x
a = quad('g',-1,x,1.e-4);

function a = g(y)
global xx
a = x^2*cos(x*y^2)
```

2. (10) Write Matlab statements to compute the product of two matrices, `A` and `B`, using the outer product formulation of summing columns of `A` times rows of `B`.

Answer:

```
[m,n]=size(A);
[n,p]=size(B);
C = zeros(m,p);
for i=1:n,
  C = C + A(:,i)*B(i,:);
end
```