

AMSC/CMSC 460 Computational Methods, Fall 2007

UNIT 1: Introduction

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AMSC/CMSC 460 Introduction

The Plan:

- What is this course about?
- What do I need to know before we begin?
- How is the course organized?

What is this course about?

What is this course about?

An introduction to scientific computing.

Our goal is to be able to

- develop mathematical models for data.
- choose appropriate algorithms for studying these models.
- understand the limitations of the models and the algorithms.

A Motivating Example

The Problem:

The speed of sound in ocean water depends on

- pressure.
- temperature.
- salinity.

These vary with depth in complicated ways.

Let z be the depth (in feet) under the ocean surface. (The z axis points downward!) Someone has measured these values for $c(z)$, the speed of sound (in ft/sec) at depth z :

$c(0) = 5042$	$c(500) = 4995$	$c(1000) = 4948$
$c(1500) = 4887$	$c(2000) = 4868$	$c(2500) = 4863$
$c(3000) = 4865$	$c(3500) = 4869$	$c(4000) = 4875$
$c(5000) = 4875$	$c(6000) = 4887$	$c(7000) = 4905$
$c(8000) = 4918$	$c(9000) = 4933$	$c(10000) = 4949$
$c(11000) = 4973$	$c(12000) = 4991$	

Since the speed of sound varies with depth, sound rays travel in curved paths. (It's like the refraction of light when you look into a fishbowl.)

We make a [mathematical model](#) for the paths using [Snell's Law](#). Suppose we broadcast a sound from a point $x = 0, z = z_0$ and let $z(x)$ be the depth of the ray when it is a horizontal distance x from the source.

Let $\theta(x)$ denote the angle between the tangent to $z(x)$ and the horizontal axis:

$$\tan \theta(x) = \frac{dz}{dx}.$$

Snell's Law says

$$\frac{\cos \theta}{c(z)} = a$$

where a is a constant.

Putting all of this information together by differentiating these equations, we can obtain the model

$$\begin{aligned} \frac{d^2 z}{dx^2} &= -\frac{c'(z)}{a^2 c(z)^3}, \\ a^2 &= \left(\frac{\cos \theta(0)}{c(z_0)} \right)^2, \\ z(0) &= z_0, \\ \frac{dz}{dx}(0) &= \tan \theta_0. \end{aligned}$$

From this model, we want, for example, to predict where the ray beginning at $z_0 = 2000$ feet at an angle of $\theta_0 = 5.4^\circ$ will go, or to find the angle corresponding to the ray that hits a given target

It turns out that this problem uses a great deal of the machinery that we will develop in this course:

- interpolating the c data with a spline.
- solution of a differential equation.
- solution of a nonlinear equation (to find the angle).

Our task: We need to choose algorithms for solving this problem, and we need to understand how good the answers from these algorithms will be.

In summary, our goal is to learn to solve mathematical modeling problems that require the use of a variety of numerical techniques.

What do I need to know before we begin?

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- We will be drawing heavily on your ability to [manipulate matrices and vectors](#):
 - addition, subtraction, multiplication of matrices and vectors.
 - solving linear systems of equations $Ax = b$.
- We will make use of some results from calculus:
 - [Taylor expansions](#)

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + \dots + \frac{h^k}{k!}f^{(k)}(x) + \frac{h^{k+1}}{(k+1)!}f^{(k+1)}(\xi),$$

where $\xi \in [x, x+h]$.

- Rolles' Theorem, MVT, etc.
- We will study solution of [ordinary differential equations](#).
- We will assume that we all know how to [program in Matlab](#). Resources:
 - There are many textbooks about learning Matlab.
 - There is an on-line tutorial: <http://www.cs.iastate.edu/~cs474/MATLAB/matlab-primer.html>
 - The Matlab `help` command is very helpful, as is the documentation at www.mathworks.com!

- Chapter 1 of Van Loan introduces a lot of Matlab capabilities: vector operations, including the “dot” commands, scripts, special fonts, xor, length, linspace, logspace, plot, semilogx, semilogy, title, xlabel, ylabel, sprintf, disp, input, ginput, axis, grid, legend, set, for, if, while, zeros, size, max, sum, close, figure, diary, hist, rand, randn, subplot, floor, fix, round, any, all, &, |, ~
- All the M-files mentioned in the text are available; see the preface of your text for the pointer to this and other useful information.
- Me, and the teaching assistant.

How is the course organized?

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See the course syllabus for information about

- topics we will cover (9 chapters of Van Loan).
- quizzes every 4 classes.
- in-class exercises.
- homeworks.
- final exam.

To get off to a good start:

- Read introductory notes on your own.
- Run some Matlab examples in Chapter 1 of your text, to make sure that you are comfortable with Matlab.

Final Words:

- Lecture notes will be available for each unit of the course.

Please print them out (preferably in color) and use them as a study aid.

The lecture notes are a work-in-progress. There are **typos** in them. I'll appreciate your help during and after class to try to find all of the errors.

- You **cannot** pass this course without doing the homework, even if you get perfect scores on quizzes and the final exam.

But you **can** pass this course if you have a passing quiz and exam average by making a reasonable effort on each homework assignment **and turning each one in on time.**