Homework4.

1. Write a Matlab program that computes the condition numbers of V and B for n = 1, . . . , 20.

```
% CMSC/AMSC 460  Fall 2007
% Homework 4
%
% Investigates the stability of polynomial interpolation
% based on the power basis and the Newton basis using the
% condition number of their coefficient matrices.
%
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% Modified by Sima Taheri, 30 Oct. 2007
%
% Parameters:
%   X: Interpolation points which are n equally
%      spaced points xi between 0 and 1.
%   n: Number of interpolation points, n = 1,...,20.
%   V: Vandermonde matrix for power basis interpolation.
%      p(x) = d_1x^n-1 + d_2x^n-2 + . . . + d_n-1x + d_n,
%   kappaV: Condition number of matrix V.
%   B: Matrix for Newton basis interpolation.
%      p(x) = c_1+c_2(x-x1)+...+c_n(x-x1)(x-x2)...(x-xn-1),
%   kappaB: Condition number of matrix B.
%
% Outputs:
%   Matrices V and B for n=4.
%   Plot of the condition numbers vs. n
%   A table of the condition numbers

nmax = 20;
for n=1:nmax,
   % Interpolation points
   X = linspace(0,1,n)';
   % Vandermonde matrix
   V = vander(X);

   % Generate B matrix
   B = zeros(n);
   B(1:n,1) = ones(n,1);
   for i=2:n
      B(i:n,i) = (X(i:n)-X(i-1)).*B(i:n,i-1);
   end

   % Condition numbers
   kappaV(n) = cond(V);
   kappaB(n) = cond(B);

   % Display the matrices V and B for n = 4
   if (n==4)
      V
      B
   end
end
```
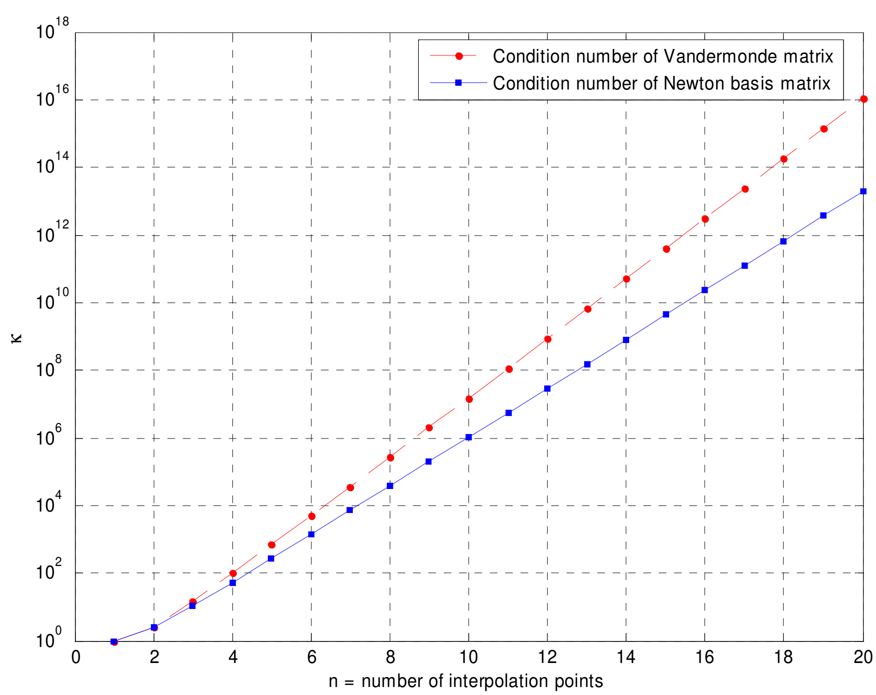

```
% Plot the condition numbers as a function of n
figure(1)
semilogy(1:nmax, kappaV, 'r.', 1:nmax, kappaB, 'b-')
legend('Condition number of Vandermonde matrix', ...
       'Condition number of Newton basis matrix')
xlabel('n = number of interpolation points')
ylabel('\kappa')

% Make a table of the condition numbers
disp('Condition numbers of Vandermonde matrix V')
disp('and Newton matrix B for various dimensions n:')
disp(' ')
disp('   n      kappa(V)    kappa(B)')
disp(sprintf('  %2d     %7.3e   %7.3e \n',[[1:nmax];kappaV; kappaB]))
-------------------------------------------------------------------

V =

         0         0         0    1.0000
    0.0370    0.1111    0.3333    1.0000
    0.2963    0.4444    0.6667    1.0000
    1.0000    1.0000    1.0000    1.0000

B =

    1.0000         0         0         0
    1.0000    0.3333         0         0
    1.0000    0.6667    0.2222         0
    1.0000    1.0000    0.6667    0.2222
```



```
Condition numbers of Vandermonde matrix V
and Newton matrix B for various dimensions n:
```

n	kappa(V)	kappa(B)
1	1.000e+000	1.000e+000
2	2.618e+000	2.618e+000
3	1.510e+001	1.106e+001
4	9.887e+001	5.307e+001
5	6.864e+002	2.659e+002
6	4.924e+003	1.361e+003
7	3.606e+004	7.057e+003
8	2.678e+005	3.689e+004
9	2.009e+006	1.940e+005
10	1.519e+007	1.025e+006
11	1.156e+008	5.431e+006
12	8.835e+008	2.886e+007
13	6.781e+009	1.537e+008
14	5.221e+010	8.201e+008
15	4.032e+011	4.382e+009
16	3.122e+012	2.344e+010
17	2.422e+013	1.256e+011
18	1.882e+014	6.731e+011
19	1.464e+015	3.612e+012
20	1.134e+016	1.939e+013

2. Explain from the condition number data why it is better to use the Newton basis rather than the power basis.

As can be seen in the above figure and also table, the condition numbers of the Newton basis matrix `B` are always less than those of the Vandermonde matrix `V` (except `n=1,2` that both matrices have the same condition numbers).
Since the accuracy of our coefficients `d` or `c` will depend on the condition number of the coefficient matrix (`V` or `B`), it is better to use the Newton basis method with smaller condition number rather than the power basis method.
Moreover, the condition number is a measure of stability or sensitivity of a matrix (or the linear system it represents) to numerical operations. In other words, we may not be able to trust the results of computations on an ill-conditioned matrix. (Matrices with small condition numbers (near 1) are said to be well-conditioned. While matrices with condition numbers much greater than one are said to be ill-conditioned.)