

An Example of Gauss Elimination

Originalmatrix:

4	4	8
2	8	7
1	3	6

1999,2001

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An Example of Gauss Elimination

Originalmatrix:

I	4	4	8
II	2	8	7
III	1	3	6

Step1:Putzerosin
firstcolumn

$-1/2I+II:$

4	4	8
0	6	3
0	2	4

$-1/4I+III:$

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An Example of Gauss Elimination

Currentmatrix:

I	4	4	8
II	0	6	3
III	0	2	4

Step2:Putzerosin
secondcolumn

$-1/3II+III:$

4	4	8
0	6	3
0	0	3

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What Gauss Elimination Accomplished:

4	4	8
2	8	7
1	3	6

→

4	4	8
0	6	3
0	0	3

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But Notice: The multipliers are important, too:

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 2 & 8 & 7 \\ \hline 1 & 3 & 6 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 1/2 & 1 & 0 \\ \hline 1/4 & 1/3 & 1 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 0 & 6 & 3 \\ \hline 0 & 0 & 3 \\ \hline \end{array}$$

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But Notice: The multipliers are important, too:

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 2 & 8 & 7 \\ \hline 1 & 3 & 6 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 1/2 & 1 & 0 \\ \hline 1/4 & 1/3 & 1 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 0 & 6 & 3 \\ \hline 0 & 0 & 3 \\ \hline \end{array}$$

A=L

U

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How this solves linear systems:

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 2 & 8 & 7 \\ \hline 1 & 3 & 6 \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline 4 \\ \hline 5 \\ \hline 5 \\ \hline \end{array}$$

Ax=b

Step1:SolveLy=b

LUx=b

Step2:SolveUx=y

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Solve $Ly = b$

$$\begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 1/2 & 1 & 0 \\ \hline 1/4 & 1/3 & 1 \\ \hline \end{array} \begin{array}{|c|} \hline y_1 \\ \hline y_2 \\ \hline y_3 \\ \hline \end{array} = \begin{array}{|c|} \hline 4 \\ \hline 5 \\ \hline 5 \\ \hline \end{array}$$

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Solve $L y = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

$$1y_1 = 4$$

$$y_1 = 4$$

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Solve $L y = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

$$1/2y_1 + y_2 = 5, y_1 = 4$$

$$y_2 = 3$$

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Solve $L y = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

$$1/4y_1 + 1/3y_2 + y_3 = 5, y_1 = 4, y_2 = 3$$

$$y_3 = 3$$

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Result: $L y = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

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Now solve $Ux = y$

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 0 & 6 & 3 \\ \hline 0 & 0 & 3 \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline 4 \\ \hline 3 \\ \hline 3 \\ \hline \end{array}$$

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Solve $Ux = y$

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 0 & 6 & 3 \\ \hline 0 & 0 & 3 \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline 4 \\ \hline 3 \\ \hline 3 \\ \hline \end{array}$$

$$3x_3 = 3$$

$$x_3 = 1$$

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Solve $Ux = y$

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 0 & 6 & 3 \\ \hline 0 & 0 & 3 \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline 4 \\ \hline 3 \\ \hline 3 \\ \hline \end{array}$$

$$6x_2 + 3x_3 = 3, x_3 = 1$$

$$x_2 = 0$$

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Solve $Ux = y$

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 8 \\ \hline 0 & 6 & 3 \\ \hline 0 & 0 & 3 \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline 4 \\ \hline 3 \\ \hline 3 \\ \hline \end{array}$$

$$4x_1 + 4x_2 + 8x_3 = 4, x_3 = 1, x_2 = 0$$

$$x_1 = -1$$

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Result: $U x = y$

$$\begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

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Result: $A x = b$

$$\begin{bmatrix} 4 & 4 & 8 \\ 2 & 8 & 7 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

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