Fall 2001 **CMSC 451: Homework 6** Samir Khuller

Due in class: Dec 6.

(1) Consider the problem DENSE SUBGRAPH: Given $G$, does it contain a subgraph $H$ that has exactly $K$ vertices and at least $Y$ edges? (The $Y$ edges must connect vertices in the subset of $K$ chosen vertices.) Prove that this problem is NP-complete.

(2) HAMILTONIAN PATH PROBLEM: given a directed graph, does it contain a path that starts at some vertex and goes to some other vertex, going through each remaining vertex exactly once.

HAMILTONIAN CYCLE PROBLEM: given a directed simple graph, does it contain a directed simple cycle that goes through each vertex exactly once.

Assume that the HAMILTONIAN PATH PROBLEM is known to be NP-complete. Given this assumption, prove that the HAMILTONIAN CYCLE PROBLEM is NP-complete for directed graphs. (Show that the HAMILTONIAN CYCLE PROBLEM is in $NP$, and is also $NP$-hard.)

(3) Assume that the following problem is $NP$-complete.
PARTITION: Given a finite set $A$ and a "size" $s(a)$ (a positive integer) for each $a \in A$. Is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$ ?

Prove that the following ZERO CYCLE problem is $NP$-complete:
Given a simple directed graph $G = (V, E)$, with positive and negative weights $w(e)$ on the edges $e \in E$. Is there a simple cycle of zero weight in $G$ ? (Hint: Reduce PARTITION to ZERO CYCLE.)

(4) Assume that there is a polynomial time algorithm for the decision version of the CLIQUE problem: given a graph $G$ and an integer $K$ the algorithm answers "T" or "F" indicating whether or not if a clique of size at least $K$ exists. Can you use this function to actually find a subset of vertices that forms the maximum clique in polynomial time? Give your algorithms running time as a function of the size of the graph and the running time of the decision algorithm. Better running times will receive a higher score.

(5) Problem 27.2-3 (pg 599). [Prob 26-2.3 (pg 663) in new edition.]