Fall 1999 **CMSC 651: Homework 1** Samir Khuller

Due in class: Sep 22.

READ Chapters 22 and 24 [CLR].

The first three homework problems are not directly related to the class material that we will cover during the first 2 weeks. I would like you to spend some time thinking about these problems to sharpen your problem solving skills. Most of the future homeworks will relate more closely to material that we cover in class.

(1) You are given $n$ nuts and $n$ matching bolts in a jumbled heap. In each operation you are allowed to take one nut and one bolt and to check if they fit. You can conclude that either you have found the matching nut for the bolt, or the nut is too big or too small. (You cannot compare two nuts or two bolts directly.) Design a "good" algorithm to find the matching pairs of nuts and bolts. How many operations does your algorithm perform?

(2) An *Euler circuit* in an undirected graph is a path that starts and ends at the same vertex, and uses every edge exactly once. Show that a connected, undirected graph has an Euler circuit if and only if every vertex has even degree. Give a linear time algorithm to find an Euler circuit (if one exists).

(3) An $n$-vertex directed graph has a *sink* if it has a vertex that has indegree $n-1$ and outdegree 0. Describe an algorithm that decides whether or not a given adjacency matrix represents a directed graph with a sink vertex by examining only $O(n)$ of the entries. Your algorithm should also run in $O(n)$ time.
(Assume that the input has already been read and is available in a matrix.)

(4) Consider the disjoint set union algorithms with path compression and union by rank. Show that any sequence of Union and Find operations, in which all Union's occur before any of the Find's runs in time proportional to the number of Union's and Find's. Assume that the UNION operations are done on the roots, so no FIND operations are being done in implementing UNION.

(5) Problem 24-1 (pg 511, CLR).