Fall 1999 **CMSC 651: Homework 3** Samir Khuller

Due in class: October 20.

(1) Let M and N be two arbitrary matchings in a bipartite graph $G = (V \cup U, E)$. Show that there exists a matching $Combine(M, N)$ in which all matched vertices of V under the matching M are matched, and all matched vertices of U under the matching N are matched.

(2) You have to broadcast n pages on a wireless channel. There are many clients interested in these pages. For each page i, each client has some "deadline" for receiving the page. For example, for page i the deadlines can be $d_i^1, d_i^2, \ldots, d_i^c$ where c is the number of clients. If a client receives the page by the deadline then it pays us 1 dollar. (Each client will pay us according to how many pages it received by the requested deadlines.) Give a polynomial time algorithm to find a broadcast schedule for the n pages to maximize the total profit.

(3) Let $G = (X, Y, E)$ be a bipartite graph. Assume for simplicity that $|X| = |Y|$.

A subset $S \subseteq (X \cup Y)$ is called a *vertex cover* if for *each* edge $e = (x, y)$ in E either $x \in S$ or $y \in S$ (or both).

Let M^* be a maximum matching in G.

(a) Prove that if S is a vertex cover then $|S| \geq |M^*|$.

(b) Give an algorithm to find the minimum cardinality vertex cover. Give a proof for its correctness.

(4) Consider a set of costs $c_{ij} \geq 0$ on the edges of a complete graph K_n with n nodes, and let $S \subseteq \{v_1, \ldots, v_n\}$. We wish to find a subgraph of K_n that has odd degree at nodes in S, an even degree (possibly zero) at all other nodes, and as low cost as possible. Show that this is a minimum weight matching problem. (Min weight matching can be solved in polynomial time in graphs that are not necessarily bipartite.)

(5) (Chinese Postman Problem) We are given a connected graph $G = (V, E)$ and a cost function $c : E \rightarrow Z^+$. We wish to find a closed walk, traversing each edge of E at least once, such that the total cost of the walk (with multiple traversals of an edge charged multiply) is as small as possible. Give a polynomial time algorithm for this problem. (You may assume a solution to problem (4) even if you did not solve problem (4).)