Fall 1999 **CMSC 651: Homework 6** Samir Khuller

Due in class: Dec 13.

(1) Given an edge weighted graph $G = (V, E)$ find a minimum weight subset of edges E' such that every vertex in G is incident on at least one edge in E'. (Such a set is called a min weight edge cover.) Give a polynomial time algorithm for solving minimum weight edge cover. If you cannot solve it for general graphs, you may solve it for bipartite graphs for partial credit. Give an argument to prove correctness of the algorithm. (You may find the following fact useful: a min weight perfect matching in any weighted graph can be found in polynomial time.)

(2) Prove that the dominating set problem is NP-complete, by DIRECTLY reducing SATISFIABILITY to it.

(3) Suppose a company has the following vehicle requirements. Jan: 230 cars, Feb: 210 cars, Mar: 440 cars, Apr: 390 cars, May: 425 cars, Jun: 450 cars. They have the following lease options. A 3 month lease for 1700\$, a 4 month lease for 2200\$, a 5 month lease for 2600\$.

Show how to formulate this problem as a linear (integer) linear programming problem, and then argue that this can be reduced to a min cost flow computation.

(4)
$$\begin{array}{ll} \max & z = 4x_1 + x_2 + 5x_3 + 3x_4 \\ \text{s.t.} & x_1 - x_2 - x_3 + 3x_4 \leq 1 \\ & 5x_1 + x_2 + 3x_3 + 8x_4 \leq 55 \\ & -x_1 + 2x_2 + 3x_3 - 5x_4 \leq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

Run the simplex algorithm to find the optimal solution to the above linear program. Also write out the Dual LP and a feasible dual solution, such that the objective function value of the dual equals the objective function value for the primal LP.

(5) Assume that we have a function $DS(G, K)$ that returns "TRUE" if and only if G has a dominating set of size K. Now use this function to design a polynomial time algorithm to actually find a subset of vertices of minimum cardinality that form a dominating set in the given graph G.