Appendix A
Proof for Lemma 2

Proof: For each link-flow pair (i_f, f), by taking squares of the following equation $Q_{i_f,f}(t+1) = Q_{i_f,f}(t) - d_{i_f,f}(t) + a_{i_f,f}(t)$, we obtain the following difference of square between the two queueing systems:

$$\begin{aligned}
&\big(Q^{R'}_{i_f,f}(t+1)\big)^2 - \big(Q^{R''}_{i_f,f}(t+1)\big)^2 \\
=&\big(Q^{R'}_{i_f,f}(t)\big)^2 - \big(Q^{R''}_{i_f,f}(t)\big)^2 + \big(d^{R'}_{i_f,f}(t)\big)^2 - \big(d^{R''}_{i_f,f}(t)\big)^2 \\
&+ \big(a^{R'}_{i_f,f}(t)\big)^2 - \big(a^{R''}_{i_f,f}(t)\big)^2 - 2\mu^{R'}_{i_f,f}(t)Q^{R'}_{i_f,f}(t) \\
&+ 2\mu^{R''}_{i_f,f}(t)Q^{R''}_{i_f,f}(t) + 2a^{R'}_{i_f,f}(t)Q^{R'}_{i_f,f}(t) \\
&- 2a^{R''}_{i_f,f}(t)Q^{R''}_{i_f,f}(t) - 2d^{R'}_{i_f,f}(t)a^{R'}_{i_f,f}(t) + 2d^{R''}_{i_f,f}(t)a^{R''}_{i_f,f}(t).
\end{aligned}$$

For any pair (f, i_f), $\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{a^{R'}_{i_f,f}(\tau)\} = \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{a^{R''}_{i_f,f}(\tau)\} = \lambda(f)$. When $i > 1$, $a_{i_f,f}(t) = d_{i-1_f,f}(t)$, $\big(a_{i_f,f}(t)\big)^2 = a_{i-1_f,f}(t)$, and $\big(d_{i_f,f}(t)\big)^2 = d_{i-1_f,f}(t)$.

Let $\overline{\mu}^{R'}_{i_f,f} \triangleq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{\mu^{R'}_{i_f,f}(\tau)\}$, and $\overline{\mu}^{R''}_{i_f,f} \triangleq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{\mu^{R''}_{i_f,f}(\tau)\}$.

Taking expectation of, time-averaging and removing equivalent terms on both sides of the above equation yields

$$\begin{aligned}
0 =& -2\big(\overline{\mu}^{R'}_{i_f,f} - \lambda(f)\big) \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{Q^{R'}_{i_f,f}(\tau)\} \\
&+ 2\big(\overline{\mu}^{R'}_{i_f,f} - \lambda(f)\big) \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{Q^{R''}_{i_f,f}(\tau)\}.
\end{aligned}$$

Since $\mathbb{E}\left\{\mu^{R'}_{i_f,f}(t)\right\} \geq \mathbb{E}\left\{\mu^{R''}_{i_f,f}(t)\right\}$, for each flow f and link i_f, at any time t, we obtain

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{Q^{R'}_{i_f,f}(\tau)\} \leq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{Q^{R''}_{i_f,f}(\tau)\},$$

which leads to the statement of the lemma. ■

Appendix B
Quantifying the Delays from Adaptive Channel Switching

One of the novel aspects of cognitive networks is adaptive channel switching; however, this can add to the delays [4]. As a specific application of our approach, we show how to estimate throughput capacity in networks with adaptive channels (*e.g.*, in cognitive networks) and end-to-end delay requirements. These constraints can be explicitly incorporated into our framework, thereby allowing us to provably quantify the trade-offs between these constraints. Much of the work dealing with these aspects (*e.g.*, [1], [3]) has only considered individual constraints such as the delay or the number of channels; our approach allows all of them to be incorporated simultaneously.

Here, we discuss a single radio interface per node; this can be easily extended to the case of multiple interfaces. Let Ψ denote the set of channels available in the system; let ψ, ψ' be two arbitrary channels in Ψ. If l and l' are incoming and outgoing links of a node respectively, let the delay in switching from channel ψ to ψ' be denoted by $d(\psi, \psi')$. Our formulation in Section VI is based on link delays, whereas switching delays are not captured because they are associated with nodes. The difficulty of applying the LP formulation lies in adapting constraints LP-(6c) and LP-(6d) to multi-channel model. We tackle this by performing a graph transformation on the network graph G to a new graph G' by the following three steps: (1) We *split* each link in G into $|\Psi|$ links, each associated with a unique channel; (2) for each node $v \in G$, we *split* it into $\big(|\mathcal{L}_{in}(v)| + |\mathcal{L}_{out}(v)|\big)|\Psi|$ nodes, each of which is incident with only one incoming or outgoing link. (3) each node incident with an incoming link is connected to each node incident with an outgoing link, by an intermediate link associated with a switch delay.

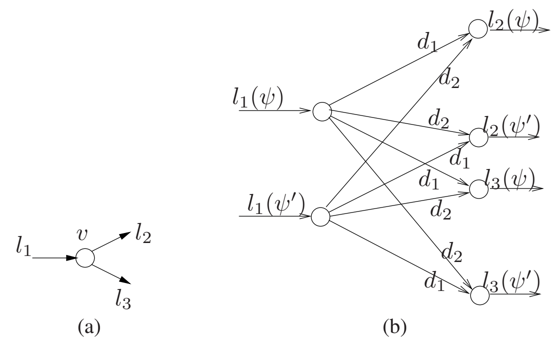


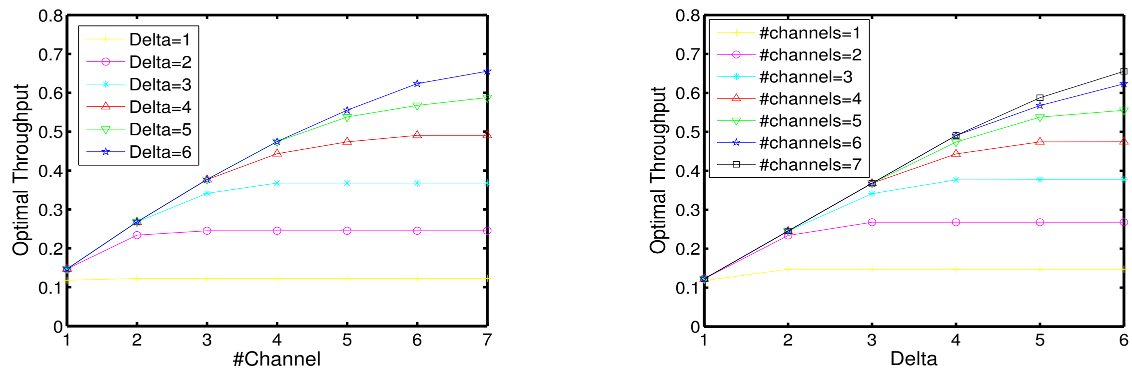
Fig. 1: (a) Node v with incoming link l_1, outgoing links l_2, l_3, and channels ψ, ψ'. (b) The reduction after node and link splitting with addition of switching link with delays d_1, d_2.

Let $l(\psi)$ denote the link associated with channel ψ in G' emerged from link l in G, Let $\mathcal{L}'$ denote the set of links in G'. Figure 7 shows an example of transforming the original network graph in Figure 7a to the graph in Figure 7b, where switch delays are $d_1 = d(\psi, \psi) = d(\psi', \psi')$ and $d_2 = d(\psi, \psi') = d(\psi', \psi)$. For node v, let $\mathcal{L}'_{(1)}(v)$ denote the new sets of links emerging from Step (1) above, which corresponds to the incoming and outgoing links in Figure 7a; and let $\mathcal{L}'_{(3)}(v)$ denote the sets of new links emerging from Step (3) above, which corresponds to the set of all the complete bipartite links in the middle connecting the incoming and outgoing links in Figure 7b.

For link $l \in G$, let $Pri(l)$ denote the primary interference set which includes all links in G sharing an end with link l. After graph transformation, for link $l(\psi) \in G'$, let $\lambda(l(\psi)) \triangleq \sum_{c\in\mathcal{C}} \lambda\left(l(\psi), c\right)$. The stability condition [2] is

$$\begin{aligned}
\lambda\left(l(\psi)\right) + \sum_{\psi'\in\Psi\setminus\{\psi\}} \lambda\left(l(\psi')\right) + \sum_{\psi'\in\Psi}\sum_{l'\in Pri(l)} \lambda\left(l'(\psi')\right) \\
+ \sum_{l'\in\mathcal{I}(l)\setminus Pri(l)} \lambda\left(l'(\psi)\right) \leq \frac{1-\epsilon}{e},\ \forall l, \forall\psi.
\end{aligned}$$

In the above inequality, note that l and l' denote links from G; the link-channel pair $l(\psi)$ denotes a link from G'. Additionally, we construct interference constraints on the intermediate switching links in G', depending on specific switching conditions. For example, when we are restricted to



(a) Optimal throughput v.s. # channels, with fixed delay bounds.

(b) Optimal throughput v.s. delay bound, with fixed # channels.

Fig. 2: Trade-offs among OPT throughput, delay, number of channels

using only one channel at a time, we can apply the following as the interference constraints for intermediate switching links:

$$\sum_{l \in \mathcal{L}'_{(1)}(v) \cup \mathcal{L}'_{(2)}(v)} \lambda(l) \leq 1, \forall v.$$

Then after modifying the flow conservation conditions and using switching delay as the link cost for any intermediate switching link, we are able to adapt the LP in Section VI to finding a multi-commodity flow vector for the multi-channel model. By using the distributed random-access scheduling scheme of [2], and setting $p(l, f)$ as in Section III, we can obtain results in the same form as Theorem 2.

A. Simulation Results for Multi-channel Networks

This complements the simulation results for single-channel networks in Section VIII. Figures 8a and 8b show the optimal throughput calculated by solving the LP's for grid topologies with 2-hop interference model on grid topologies. As expected, the total throughput increases as additional channels are equipped and delay bounds are relaxed. Saturation points are observed in both plots. Addition of channel resources alleviates the interference, thus yielding a slower saturation process. Also, loosening the delay bound produces similar effects, and the addition of channels make the optimization process to exploit more under the delay bounds.

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