This homework should be scanned/imaged and uploaded to Elms, or optionally turned into me in class. To get full credit, you must print this homework and write your solutions in the space provided. Use either my slides, or Boyd §3.2 as a reference.

1. (20 points) (a) Give an example of a *convex* function that is not proper.

   (b) Give an example of a *convex* function that is bounded below, but has no minimizer.

   (c) Give an example of a convex *function* that is not closed. Hint: Suppose I gave you a convex function that holds beer. What could you do to make all the beer leak out?

   (d) Give an example of a *convex* function that has unbounded level sets, but has at least one minimizer.

2. (50 points) For this problem, you may use any of the results/rules from the lecture slides.

   (a) Consider the set of probability functions $\{f | f \geq 0, \text{and} \int f = 1\}$. Is this a convex set?

   (b) Consider the function $M_5 : \mathbb{R}^{10} \rightarrow: \mathbb{R}$, where $M_5(z)$ returns the sum of the 5 largest entries in x. Is this function convex? Why?

   (c) Consider the set of low-rank matrices $\{A | \text{rank}(A) < k\}$. Is this set convex? Why?

(d) Let $C \in \mathbb{R}^n$ be a compact (closed and bounded) convex set. For a scalar x, let C(x) be the set of all points in C whose first coordinate is closer to x than any other point. More precisely: $C(x) = \{a \in C \,|\, \forall b \in C, |a_1 - x| \leq |b_1 - x|\}$. Is $C(x)$ a convex set for any choice of C and x? Why?

(e) Is the set $C = \{(x, y) \,|\, \|x\| \leq y\}$ convex? Is it a cone? Why? What about the set $C = \{(x, y) \,|\, \|x\|^2 \leq y\}$? Is this convex? Is this a cone?

3. (10 points) Suppose f is strongly convex. Show that the gradient of f is *strongly monotone,* i.e., there is a constant $m > 0$ with

$$\langle x - y, \nabla f(x) - \nabla f(y) \rangle \geq m\|x - y\|^2.$$

4. (20 points) Suppose that $g(x)$ is convex and $h(x)$ is concave (i.e. $-h(x)$ is convex). Suppose we restrict both functions into a closed, convex set C such that both $g(x)$ and $h(x)$ are always positive when $x \in C$. Prove that the function $f(x) = g(x)/h(x)$ is quasi-convex. (note: It follows that every local minimum is also global)