

Your homework submission must contain the following to receive full credit:

- A single script called “hmwk1.m” or “hmwk1.py” that requires no arguments. This script should print required outputs to the console. Required outputs will be stated in bold in the problems below. Each output should be labeled – i.e., print some text above each output to clearly state what it is.
 - A pdf called hmwk1_results.pdf with your solutions to all theory problems and also the text output from the hmwk1 script (you may want to use the *verbatim* package in latex to copy and paste the results into a document). See the posted homework example.
 - As many other code files as you wish - but only things you wrote or solutions to previous homeworks.
1. Create an 8×8 matrix H using the command `hilb(8)` in Matlab, or `scipy.linalg.hilbert(8)` in Python. Generate a random vector x , and compute $Hx = b$. Add a tiny amount of noise to b . Then recover x from b by running the command `xhat = inv(H)*b`.

How accurate is the recovered x ? Why did this happen? You don't need to provide any code or console output, just describe what you did and what you got in a few sentence.

2. The dual norm of $\|\cdot\|$ is defined as

$$\|x\|_* = \max_{\|z\| \leq 1} z^T x.$$

Prove the dual norm is indeed a norm.

3. Suppose we want to recover the solution to the system

$$Ax = b.$$

We don't know b exactly, but we have a noisy measurement vector \hat{b} . To do this, we could compute $\hat{x} = A^{-1}\hat{b}$. Prove that

$$\frac{\|x - \hat{x}\|}{\|x\|} \leq \kappa \frac{\|b - \hat{b}\|}{\|b\|},$$

where κ is the condition number of A .

4. Consider the measurement model

$$y = Dx + \eta$$

where $D \in m \times n$ is a measurement matrix, and $\eta \in \mathbb{R}^m$ is a noise vector with distribution

$$\eta \sim \frac{1}{\sqrt{(2\pi)^m |\Sigma|}} \exp\left(-\frac{1}{2} \eta^T \Sigma^{-1} \eta\right)$$

for some covariance matrix Σ , where $|\Sigma|$ denotes the determinant of Σ . Suppose we have prior knowledge that *each* entry in x is drawn from an i.i.d. Laplace distribution

$$x_i \sim \frac{1}{2b} \exp(-|x|/b).$$

Derive the negative log-likelihood function for x given y . Write the complete NLL without throwing away any constants (although you may use Bayes' rule, which implicitly throws away a normalization constant). THEN, throw away constants and write a minimization problem for finding the maximum likelihood estimator (this final answer should look quite simple).

5. Write a function with signature

```
M = buildmat(m, n, condNumber)
```

that returns an $m \times n$ matrix with condition number `condNumber`. Your matrix must be **dense**. Write a few sentences

Test your function by creating a 3×5 matrix with condition number 2, and computing the condition number using the `cond` function in matlab/numpy. **Print the matrix and the computed condition number to the console. Repeat for a 5×3 matrix, and a 5×5 matrix.**